# THE KING'S SCHOOL

**2010**
**Higher School Certificate**
# Trial Examination

# Mathematics Extension 1

**General Instructions**

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

**Total marks – 84**

- Attempt Questions 1-7
- All questions are of equal value

**Disclaimer**
This is a Trial Higher School Certificate Examination only. Whilst it reflects and mirrors both the format and topics of the Higher School Certificate Examination designed by the NSW Board of Studies for the respective sections, there is no guarantee that the content of this examination exactly replicates the actual Higher School Certificate Examination.

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**Total marks – 84**
**Attempt Questions 1-7**
**All questions are of equal value**

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

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**Question 1 (12 marks)** Use a SEPARATE writing booklet. **Marks**

(a) Find $\int 2\cos^2 x \, dx$ **2**

(b) When $P(x) = x^4 + Ax + 2A$ is divided by $x + 1$, the remainder is $-1$.

Find the value of $A$. **2**

(c) The gradients of two lines are $\sqrt{2} + 1$ and $\sqrt{2} - 1$

Find the acute angle between the lines. **2**

(d) Solve the inequality $\dfrac{3x - 1}{x + 2} < 3$ **3**

(e) $f(x) = 2x - 7\ln(1 + x) = 0$ has a root near $x = 7$.

Use Newton's method once to find a one decimal place approximation to this root. **3**

**End of Question 1**

**Question 2 (12 marks)** Use a SEPARATE writing booklet. **Marks**

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(a) Find the positive integer $n$ for which

$357 - (1.05 + 1.05^2 + 1.05^3 + \ldots + 1.05^n)$ is approximately equal to zero. **3**

(b) Evaluate $\displaystyle\int_{-10}^{10} \frac{40}{25 + 16x^2}\, dx$

Give your answer in simplest exact form. **4**

(c) (i) Show that the derivative of $e^{-x}(x + 1)$ is $-xe^{-x}$ **1**

(ii) Use the substitution $u = \ln x$ to evaluate $\displaystyle\int_{1}^{e} \frac{\ln x}{x^2}\, dx$ **4**

**End of Question 2**

**Question 3 (12 marks)** Use a SEPARATE writing booklet. **Marks**

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(a) Show that $\lim_{x \to 0} \dfrac{\sin 7x}{\sin 4x} = \dfrac{7}{4}$ **2**

(b) Let $f(n) = \left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\ldots\left(1 + \frac{1}{n-1}\right)\left(1 + \frac{1}{n}\right)$

(i) Prove by mathematical induction for positive integers $n$ that $f(n) = n + 1$ **3**

(ii) Show that $f(n) = n + 1$ without using induction. **1**

(c) Let $f(x) = -\sin^{-1}(2x - 1)$

(i) Find the domain of the function. **1**

(ii) Sketch the graph of $f(x) = -\sin^{-1}(2x - 1)$ **2**

(iii) Find, in simplest form, the inverse function $f^{-1}(x)$ and state its domain and range. **3**

**End of Question 3**

**Question 4 (12 marks)** Use a SEPARATE writing booklet. **Marks**

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(a) A particle is moving in simple harmonic motion on the $x$ axis with acceleration equation of motion given by $\ddot{x} = -9(x + 1)$

The particle is initially at the origin with velocity $3\sqrt{3}$

(i) State the period of the motion. **1**

(ii) By integration, prove that $v^2 = 9(4 - (x + 1)^2)$, where $v$ is the velocity. **2**

(iii) Find the amplitude $A$ of the motion. **1**

(iv) The displacement equation of motion is given by

$$x = -1 + A\cos(nt + \alpha), \text{ where } n > 0 \text{ and } -\frac{\pi}{2} < \alpha \leq \frac{\pi}{2}$$

Find $\alpha$ **2**

(b) (i) Express $\sin\theta$ in terms of $t = \tan\frac{\theta}{2}$ **1**

(ii) Solve the equation $\sin\theta + 2\cos\theta + 2 = 0,\ 0 < \theta < 2\pi$

Give your solutions correct to two decimal places where appropriate. **3**

(c) Let $f(x) = \cos^{-1}\left(\frac{1}{x}\right)$

By finding $f'(x)$, or otherwise, show that $f(x)$ is an increasing function for all values of $x$ in its domain. **2**

**End of Question 4**

**Question 5 (12 marks)** Use a SEPARATE writing booklet. **Marks**

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(a) A spherical balloon is deflating at a rate given by $\frac{dV}{dt} = 4\pi kr$ , where $V$ is the volume and $r$ is the radius at any time $t$. $k$ is a constant. The balloon remains spherical at all times so that $V = \frac{4}{3}\pi r^3$

(i) Prove that $\frac{dr}{dt} = \frac{k}{r}$ **3**

(ii) The balloon initially had a volume of $2304\pi \text{ cm}^3$ and it was deflated completely after 6 seconds.

Find the value of $k$. **4**

(b) Solve $\binom{n}{2} = n$ **1**

(c) Find the term independent of $x$ in the expansion of $2x^2\left(x + \frac{2}{x^2}\right)^{19}$ **4**

**End of Question 5**

**Question 6 (12 marks)** Use a SEPARATE writing booklet. **Marks**

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(a) A body with temperature $40^\circ$ C is placed in a room of constant surrounding temperature $20^\circ$ C.

It is being heated at a constant rate of $1.6^\circ$ C per second and is simultaneously being cooled according to Newton's law of cooling.

Therefore, its temperature $T^\circ$ C at any time $t$ seconds can be determined by

$$\frac{dT}{dt} = 1.6 + k(T - 20) \quad , \quad \text{where } k \text{ is a constant.}$$

When the body reaches a temperature of $100^\circ$ C it remains at that temperature.

(i) Show that $k = -0.02$ **1**

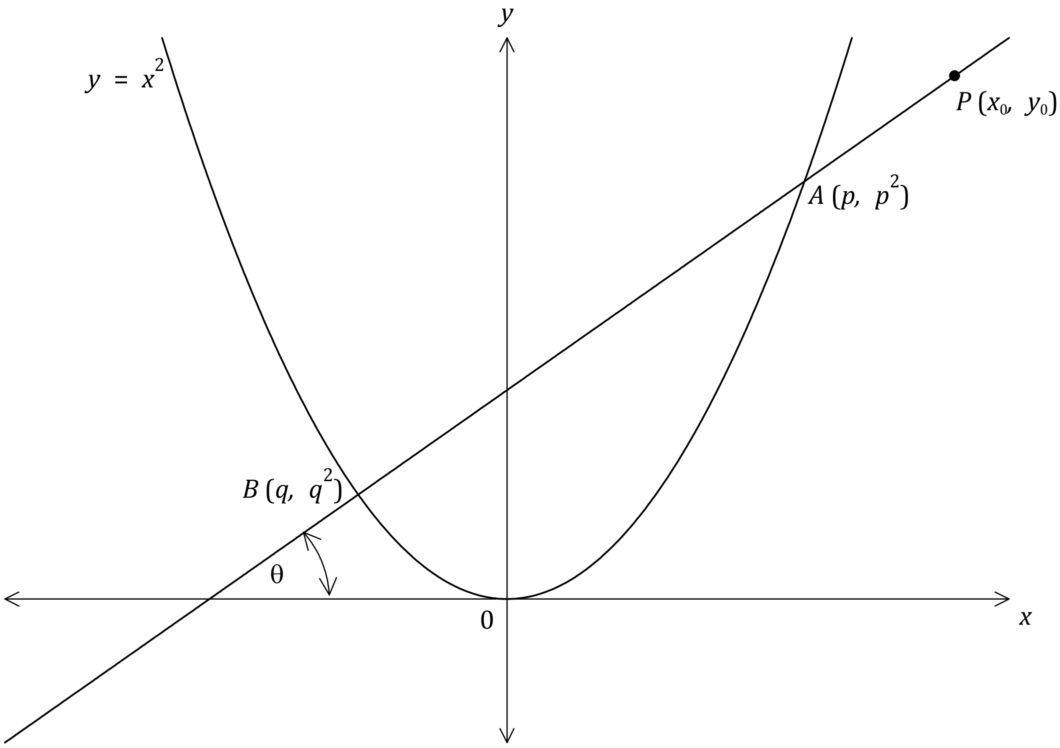
(ii) Deduce that $\frac{dT}{dt} = -0.02(T - 100)$ **1**

(iii) Verify that $T = 100 + Ae^{-0.02t}$ satisfies the differential equation in (ii). **1**

(iv) Find, correct to the nearest second, when the temperature of the body is $80^\circ$ C **3**

**Question 6 continues on the next page**

(b)




$A(p, p^2)$ and $B(q, q^2)$ are two points on the parabola $y = x^2$. The chord $AB$ makes the acute angle $\theta$ with the $x$ axis as in the diagram.

$P(x_0, y_0)$ is a point on $BA$ produced as in the diagram.

(i) Show that $\tan\theta = p + q$ **1**

(ii) By finding the equation of the chord $AB$, show that $y_0 = (p + q)x_0 - pq$ **2**

(iii) Show that $\text{PA} = \dfrac{x_0 - p}{\cos\theta}$ **1**

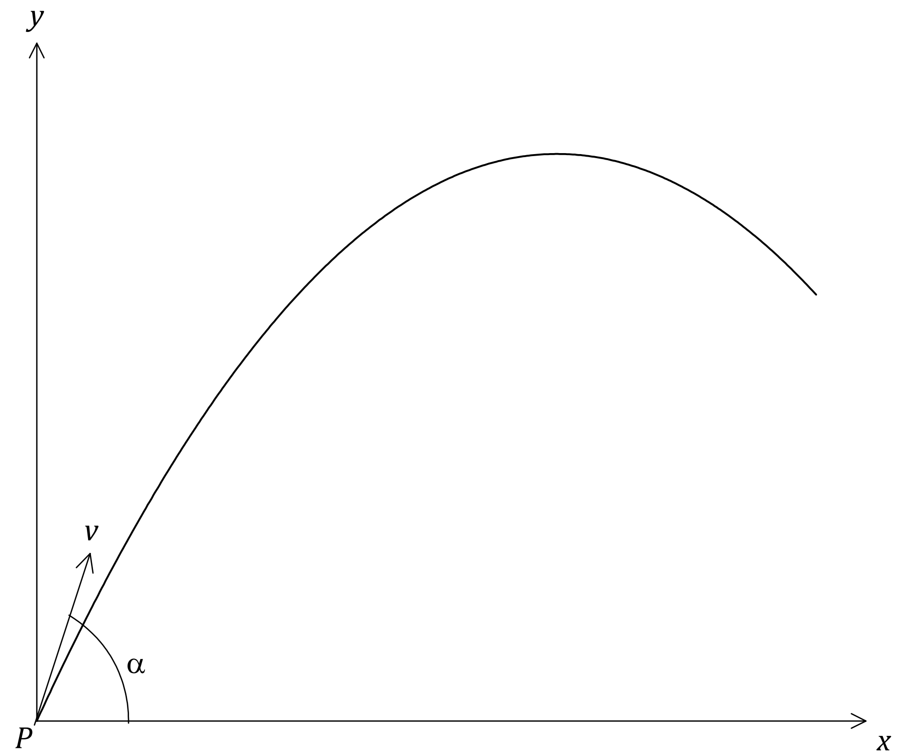
(iv) Similarly, $\text{PB} = \dfrac{x_0 - q}{\cos\theta}$ **[ DO NOT SHOW THIS ]**

Deduce that $\text{PA} \times \text{PB} = (x_0^2 - y_0)(1 + (p + q)^2)$ **2**

**End of Question 6**

**Question 7 (12 marks)** Use a SEPARATE writing booklet. **Marks**

(a)




A particle is fired from $P(0,\ 0)$ with speed $v$ at an elevation of $\alpha$ .

$g$ is the acceleration due to gravity.

The equations of motion are given by

$\ddot{x} = 0$ $\qquad\qquad$ $\ddot{y} = -g$

$\dot{x} = v\cos\alpha$ $\qquad\qquad$ $\dot{y} = -gt + v\sin\alpha$

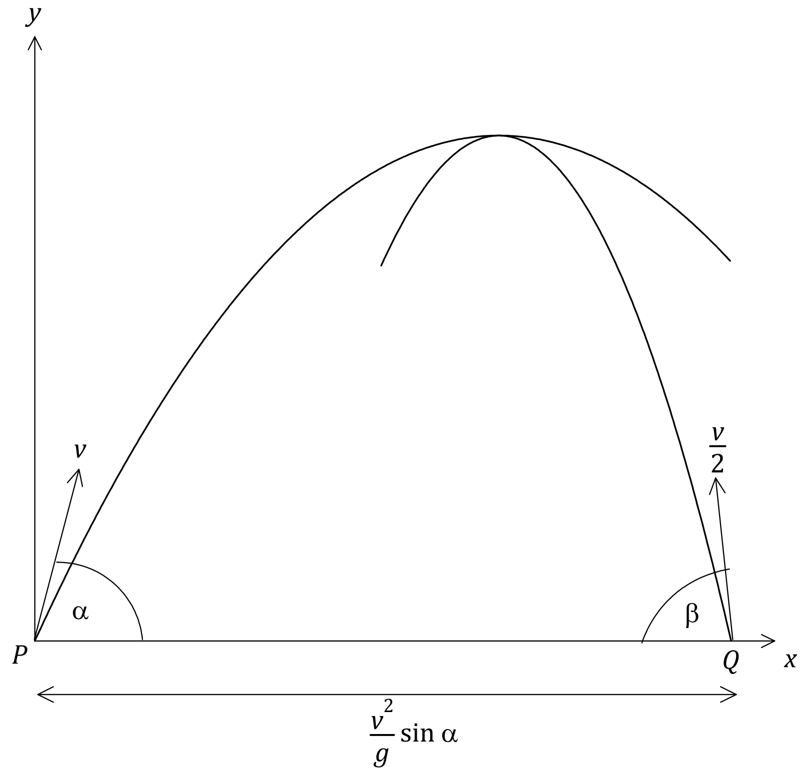
$x = v\cos\alpha\, t$ $\qquad\qquad$ $y = -g\dfrac{t^2}{2} + v\sin\alpha\, t$

**[ DO NOT PROVE THESE ]**

Let the particle reach its greatest height when $t = T$

(i) Show that $T = \dfrac{v\sin\alpha}{g}$ **1**

**Question 7 continues on the next page**




At the same time as the particle from $P$ is fired, another particle is fired from $Q$ where $PQ = \frac{v^2}{g}\sin\alpha$

The particles travel toward each other in the same vertical plane and collide at the instant when they both reach their greatest height.

The equations of motion of the particle from $Q$ are given by

$$\ddot{x} = 0 \qquad \ddot{y} = -g$$

$$\dot{x} = -\frac{v}{2}\cos\beta \qquad \dot{y} = -gt + \frac{v}{2}\sin\beta$$

$$x = \frac{-v}{2}\cos\beta\, t + \frac{v^2}{g}\sin\alpha \qquad y = -g\frac{t^2}{2} + \frac{v}{2}\sin\beta\, t$$

**[ DO NOT PROVE THESE ]**

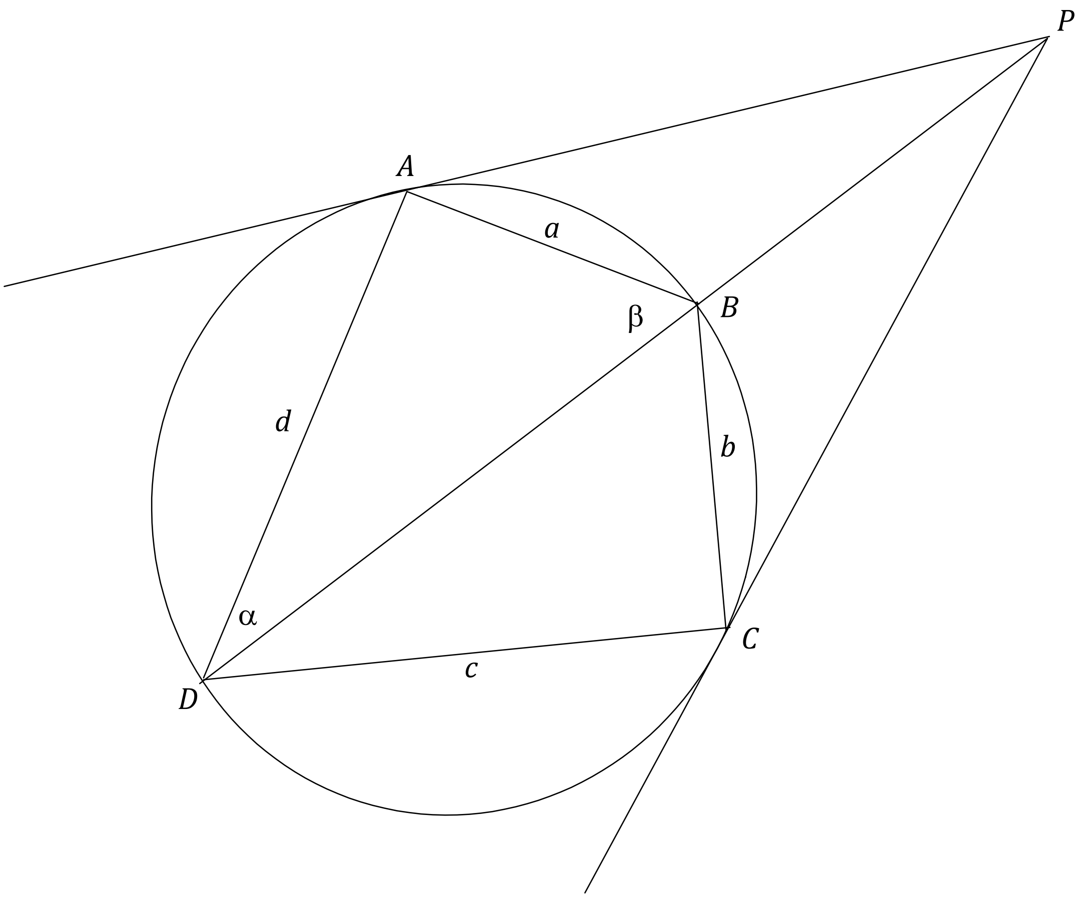
(ii) Prove that $2\sin\alpha = \sin\beta$ **1**

(iii) Prove that $2\cos\alpha = 2 - \cos\beta$ **2**

(iv) Find the value of $\cos\beta$ **2**

**Question 7 continues on the next page**

(b)




Tangents are drawn from $P$ to touch the circle at $A$ and $C$. From $P$, a secant is drawn to meet the circle at $B$ and $D$.

Let $AB = a,\ BC = b,\ CD = c,\ DA = d$

Let $\angle ADB = \alpha\,,\ \angle ABD = \beta$

(i) Explain why $\angle PAB = \alpha$ **1**

(ii) Use the sine rule in $\Delta ABD$ and $\Delta ABP$ to show that $\frac{a}{d} = \frac{PB}{PA}$ **3**

(iii) Deduce that $ac = bd$ **2**

**End of Examination Paper**

## Standard Integrals

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note: $\ln x = \log_e x, \quad x > 0$

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# Mathematics Extension 1

| Question | Algebra and Number | Geometry | Functions | Trigonometry | Differential Calculus | Integral Calculus | Total |
|---|---|---|---|---|---|---|---|
| 1 | (b), (c), (d) 7 | | | | (e) **3** | (a) **2** | **12** |
| 2 | (a) **3** | | | | (c)(i) **1** | (b), (c)(ii) **8** | **12** |
| 3 | (b) **4** | | (a), (c) **8** | | | | **12** |
| 4 | | | (a) (i), (iii) **2** | (a)(iv) (b) **6** | (c) 2 | (a)(ii) **2** | **12** |
| 5 | (b), (c) **5** | | | | (a)(i) **3** | (a)(ii) **4** | **12** |
| 6 | (b)(i), (ii) **3** | | (a)(i), (ii), (iv) **5** | (b) (iii), (iv) **3** | (a)(iii) **1** | | **12** |
| 7 | | (b) **6** | (a)(i) (ii), (iii) **4** | (a)(iv) 2 | | | **12** |
| Total | **22** | **6** | **19** | **11** | **10** | **16** | **84** |

# TKS EXTENSION 1 TRIAL 2010 SOLUTIONS

**Qn 1** (a) $I = \int \cos 2x + 1 \, dx = \frac{1}{2}\sin 2x + x \quad (+c)$

(b) $P(-1) = 1 - A + 2A = -1 \qquad \therefore A = -2$

(c) $\tan\alpha = \dfrac{\sqrt{2}+1-(\sqrt{2}-1)}{1+(\sqrt{2}+1)(\sqrt{2}-1)} = \dfrac{2}{1+1} = 1$

$\Rightarrow \alpha = \frac{\pi}{4}$

(d) (LOTS OF WAYS)

If $x+2>0$ i.e. $x>-2$ then $3x-1 < 3x+6$

$\therefore x>-2$ is a solution

Clearly, $x \not< -2$

$\therefore x>-2$ is the solution

(e) $f'(x) = 2 - \dfrac{7}{1+x}$

$\therefore x_1 = 7 - \dfrac{14 - 7\ln 8}{2 - \frac{7}{8}} = 7.49\ldots$

$= 7.5$, 1 d.p.

Question 2

(a) $$357 - \frac{1.05(1.05^n - 1)}{.05} = 0$$

$$\Rightarrow 1.05^n = \frac{357 \times .05}{1.05} + 1 = 18$$

$$\therefore n \ln 1.05 = \ln 18$$

$$n = \frac{\ln 18}{\ln 1.05} = 59.24\ldots$$

$$\Rightarrow n = 59$$

(b) $$I = 2\int_0^{10} \frac{40}{5^2 + (4x)^2}\,dx$$

$$= 80\ \frac{1}{5}\cdot\frac{1}{4}\left[\tan^{-1}\frac{4x}{5}\right]_0^{10}$$

$$= 4\tan^{-1} 8$$

(c) (i) $$\frac{d}{dx} e^{-x}(x+1) = e^{-x}.1 + (x+1)(-e^{-x})$$

$$= -e^{-x}(x+1-1)$$

$$= -xe^{-x}$$

(ii) $u = \ln x$ ; $x = 1,\ u = 0$

$\frac{du}{dx} = \frac{1}{x}$ ; $x = e,\ u = 1$

$$\therefore I = \int_0^1 \frac{u}{x}\,dx = \int_0^1 \frac{u}{e^u}\,du$$

$$= \int_0^1 ue^{-u}\,du$$

$$= -\left[e^{-u}(u+1)\right]_0^1 \text{ from (i)}$$

$$= -(2e^{-1} - 1) = 1 - 2e^{-1}$$

## Question 3

(a) $= \lim_{x\to 0} \frac{\sin 7x}{7x} \cdot \frac{4x}{\sin 4x} \cdot \frac{7}{4}$

$= 1 \times 1 \times \frac{7}{4} = \frac{7}{4}$

(b) (i) $f(1) = 1 + \frac{1}{1} = 2 = 1 + 1$

$\therefore$ Assume $f(n) = n+1$ for some integer $n \geq 1$

Then $f(n+1) = \left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right) \cdots \left(1+\frac{1}{n}\right)\left(1+\frac{1}{n+1}\right)$

$= (n+1)\left(1+\frac{1}{n+1}\right)$, using the assumption

$= n+1+1 = n+2$

$\therefore$ by induction, it's true

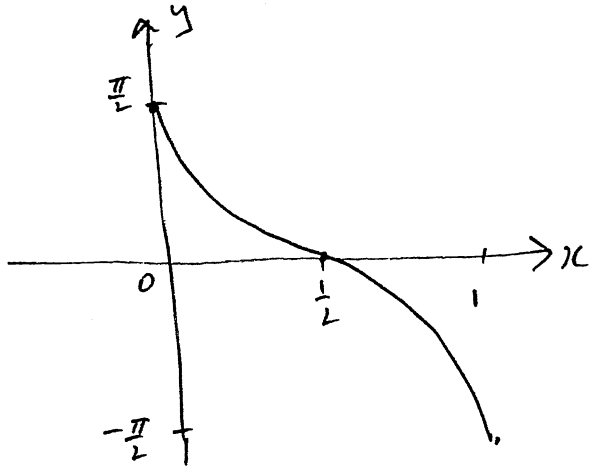
(ii) $f(n) = 2 \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdots \cdot \frac{n}{n-1} \cdot \frac{n+1}{n}$

$= n+1$

(c) (i) $-1 \leq 2x - 1 \leq 1$

$\therefore 0 \leq 2x \leq 2 \Rightarrow$ domain is $0 \leq x \leq 1$

(ii)




(iii) $f^{-1}$: $x = -\sin^{-1}(2y-1)$

or $\sin^{-1}(2y-1) = -x$

$\therefore 2y - 1 = \sin(-x) = -\sin x$

$\Rightarrow y = \frac{1-\sin x}{2}$

with domain $|x| \leq \frac{\pi}{2}$

& range $0 \leq y \leq 1$

## Question 4

(a) (i) $n^2 = 9 \Rightarrow$ period $T = \frac{2\pi}{3}$

(ii) $\frac{d(\frac{1}{2}v^2)}{dx} = -9(x+1)$

$\therefore \frac{1}{2}v^2 = -9\frac{(x+1)^2}{2} + k$

$: \frac{1}{2} \cdot 27 = -\frac{9}{2} + k \Rightarrow k = \frac{36}{2}$

$\therefore v^2 = -9(x+1)^2 + 36 = 9(4 - (x+1)^2)$

(iii) $A^2 = 4 \Rightarrow A = 2$ OR $v = 0 \Rightarrow (x+1)^2 = 4$

$\Rightarrow x + 1 = 2$ or $-2$

$x = 1$ or $-3$

$\therefore 2A = 4, \ A = 2$

(iv) $x = -1 + 2\cos(3t + \alpha)$

$\dot{x} = -6\sin(3t + \alpha)$

$\Rightarrow 0 = -1 + 2\cos\alpha$, $\cos\alpha = \frac{1}{2}$ $\Rightarrow \alpha = -\frac{\pi}{3}$

and $3\sqrt{3} = -6\sin\alpha$, $\sin\alpha = -\frac{\sqrt{3}}{2}$

(b) (i) $\sin\theta = \frac{2t}{1+t^2}$

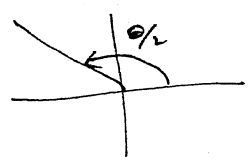
(ii) Put $t = \tan\frac{\theta}{2}$, $0 < \frac{\theta}{2} < \pi$

Check $\theta = \pi \Rightarrow 0 - 2 + 2 = 0$ $\therefore$ $\theta = \pi$ is a solution

Next $\frac{2t}{1+t^2} + 2\frac{1-t^2}{1+t^2} + 2 = 0$

$\therefore t + 1 - t^2 + 1 + t^2 = 0$

$\Rightarrow t = \tan\frac{\theta}{2} = -2$ $\Rightarrow$




$\therefore \frac{\theta}{2} = \pi - \tan^{-1}2$

$\therefore \theta = 4.07$, 2 d.p.

(c) $f(x) = \cos^{-1}(x^{-1})$

$$f'(x) = -\frac{1}{\sqrt{1-\frac{1}{x^2}}} \cdot -x^{-2} = \frac{1}{x^2\sqrt{1-\frac{1}{x^2}}}$$

$> 0 \quad \forall x$ in domain

$\therefore f(x)$ increases $\forall x$ in domain

Question 5

(a) (i) $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$

$\therefore \frac{dr}{dt} = \frac{dr}{dV} \cdot \frac{dV}{dt}$

$= \frac{1}{4\pi r^2} \cdot 4\pi k r = \frac{k}{r}$

(ii) $t=0,\quad V = 2304\pi = \frac{4}{3}\pi r^3$

$\Rightarrow r^3 = \frac{2304 \times 3}{4} \quad \therefore r = 12$

Now $\frac{dt}{dr} = \frac{r}{k}$

$\therefore kt = \frac{r^2}{2} + c$

$\therefore 0 = \frac{12^2}{2} + c,\quad c = -72$

$\therefore kt = \frac{r^2}{2} - 72$

$t = 6,\ r = 0 \quad \Rightarrow 6k = -72$

$k = -12$

(b) $\frac{n!}{(n-2)!\,2} = n \Rightarrow \frac{n(n-1)}{2} = n$

$\therefore \frac{n-1}{2} = 1$ since $n \neq 0$

$\therefore n = 3$

(c) For $\left(x + \frac{2}{x^2}\right)^{19}$, $u_{k+1} = \binom{19}{k} x^{19-k} \left(\frac{2}{x^2}\right)^k$

$= \binom{19}{k} 2^k \frac{x^{19-k}}{x^{2k}}$

$= \binom{19}{k} 2^k x^{19-3k}$

$\therefore$ For term indept of $x$ we need $19-3k = -2$

or $3k = 21$

$k = 7$

$\therefore$ term is $2 \cdot \binom{19}{7} 2^7 = \binom{19}{7} 2^8$

## Question 6

(a) (i) $T=100$, $\frac{dT}{dt}=0$

$$\Rightarrow \quad 0 = 1.6 + k(100-20)$$

$$k = \frac{-1.6}{80} = -0.02$$

(ii) $\therefore \frac{dT}{dt} = 1.6 - 0.02(T-20)$

$$= 0.02 \times 20 + 1.6 - 0.02T$$

$$= 2 - 0.02T$$

$$= -0.02(T-100)$$

(iii) If $T = 100 + Ae^{-0.02t}$

then $\frac{dT}{dt} = A(-0.02e^{-0.02t})$

$$= -0.02(Ae^{-0.02t})$$

$$= -0.02(T-100)$$

(iv) $t=0$, $T=40$

$$\Rightarrow 40 = 100 + A, \quad A = -60$$

$$\therefore T = 100 - 60e^{-0.02t}$$

$T = 80 \Rightarrow -60e^{-0.02t} = -20$

or $e^{-0.02t} = \frac{1}{3}$

or $e^{0.02t} = 3$

$\therefore 0.02t = \ln 3$

$$t = \frac{\ln 3}{.02} = 54.93\ldots$$

$= 55$ s, nearest second

(b) (i) $\tan\theta$ is the gradient of line AB

$$= \frac{p^2 - q^2}{p - q} = \frac{(p-q)(p+q)}{p-q} = p + q$$

(ii) Equation of AB is $y - p^2 = (p+q)(x-p)$

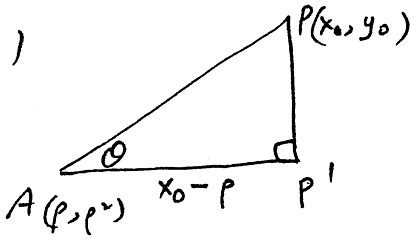
i.e. $y = (p+q)x - p^2 - pq + p^2$

or $y = (p+q)x - pq$

But $P(x_0, y_0)$ is on AB

$\therefore \; y_0 = (p+q)x_0 - pq$

(iii)




$\Rightarrow \cos\theta = \dfrac{x_0 - p}{PA}$

or $PA = \dfrac{x_0 - p}{\cos\theta}$

(iv) Similarly, $PB = \dfrac{x_0 - q}{\cos\theta}$

$$\therefore PA \cdot PB = \frac{(x_0 - p)(x_0 - q)}{\cos^2\theta}$$

$$= \frac{x_0^2 - (p+q)x_0 + pq}{\cos^2\theta}$$

$$= \frac{x_0^2 - ((p+q)x_0 - pq)}{\cos^2\theta}$$

$$= \frac{x_0^2 - y_0}{\cos^2\theta} \quad \text{from (ii)}$$

$= (x_0^2 - y_0)\sec^2\theta = (x_0^2 - y_0)(1 + \tan^2\theta)$

$= (x_0^2 - y_0)(1 + (p+q)^2)$ from (i)

## Question 7

(i) $\dot{y} = 0 \Rightarrow -gT + v\sin\dot{\alpha} = 0$

$$T = \frac{v\sin\dot{\alpha}}{g}$$

(ii) From (i), $\frac{v}{2}\sin\dot{\beta} = v\sin\dot{\alpha}$

$$\Rightarrow 2\sin\dot{\alpha} = \sin\beta$$

(iii) at $t = T$,

$$v\cos\alpha\, T = -\frac{v}{2}\cos\beta\, T + \frac{v^2}{g}\sin\dot{\alpha}$$

$$\Rightarrow v\cos\alpha\, T = -\frac{v}{2}\cos\beta\, T + vT \quad \text{from (i)}$$

$$\therefore \cos\alpha = -\frac{\cos\beta}{2} + 1$$

$$\text{or } 2\cos\alpha = 2 - \cos\beta$$

(iv) From (ii) & (iii),

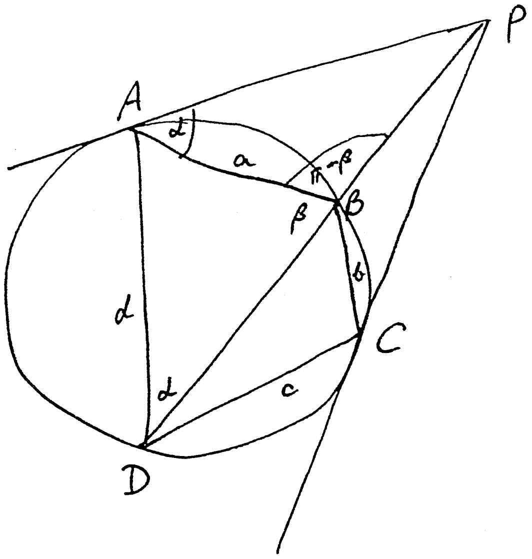
$$4(\sin^2\dot{\alpha} + \cos^2\alpha) = \sin^2\beta + 4 - 4\cos\beta + \cos^2\beta$$

$$\Rightarrow 4 = 1 + 4 - 4\cos\beta$$

$$\therefore 4\cos\beta = 1$$

$$\text{or } \cos\beta = \frac{1}{4}$$

(b) (i)




$\angle PAB = \angle ADB = \alpha$, alt. seg thm.

(ii) $\angle PBA = \pi - \beta$ , $\angle DBP$ a st. $\angle$

$\therefore$ In $\Delta ADB$, $\dfrac{a}{d} = \dfrac{\sin\alpha}{\sin\beta}$

& in $\Delta APB$, $\dfrac{PB}{PA} = \dfrac{\sin\alpha}{\sin(\pi-\beta)} = \dfrac{\sin\alpha}{\sin\beta}$

$\therefore \dfrac{a}{d} = \dfrac{PB}{PA}$

(iii) From (ii), similarly we have $\dfrac{b}{c} = \dfrac{PB}{PC}$

But $PA = PC$, lengths of tangents to circle

$\therefore \dfrac{a}{d} = \dfrac{b}{c}$ from (ii)

ie. $ac = bd$